Errata for "Rényi Divergence and Kullback-Leibler Divergence"

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Theorem 1: Upon trying to formalize the proof from the paper in the Lean theorem prover, Rémy Degenne noticed that the proof for the case $0 < \alpha < 1$ is incomplete. As in the paper, let $P_{|\mathcal{G}}$ denote the restriction of a measure P to sub-σ-algebra $\mathcal{G}$, and let $\tilde{P}$ be the absolutely continuous component of P with respect to Q. Then to complete the proof we need to observe that

$$\frac{\mathrm{d}\tilde{P}_{|\mathcal{G}}}{\mathrm{d}Q_{|\mathcal{G}}} \leq \frac{\mathrm{d}P_{|\mathcal{G}}}{\mathrm{d}Q_{|\mathcal{G}}} \qquad (Q\text{-a.s.}), \tag{1}$$

which implies that

$$\frac{1}{\alpha - 1} \ln \int \left(\frac{\mathrm{d}P_{|\mathcal{G}}}{\mathrm{d}Q_{|\mathcal{G}}}\right)^{\alpha} \mathrm{d}Q \leq \frac{1}{\alpha - 1} \ln \int \left(\frac{\mathrm{d}\tilde{P}_{|\mathcal{G}}}{\mathrm{d}Q_{|\mathcal{G}}}\right)^{\alpha} \mathrm{d}Q.$$

As Rémy pointed out to us, equality in (1) need not hold in general.

Lemma 2: The lower bound in the lemma is incorrect, and should be replaced by

$$(x-1)(2-x) \leq \ln x \qquad \text{for } x > 1/2.$$

See e.g. Cesa-Bianchi et al. [2007, Lemma 1] for a proof. This does not make any difference for the proof of Theorem 5, which is the only place where Lemma 2 is used.

References

N. Cesa-Bianchi, Y. Mansour, and G. Stoltz. Improved second-order bounds for prediction with expert advice. *Machine Learning*, 66(2/3):321–352, 2007.